

M. Ahmadi

The title of my talk is "Baer-type ring characterizations of Leavitt path algebras"

**Masoud Amini**

**The legacy of Irving Kaplansky (1917-2006)**

Massoud Amini (Tarbiat Modares University)

Abstract: Irving Kaplansky (known to his closed friends as Kap) was born in Toronto shortly after his parents emigrated to Canada from Poland. His mathematical legacy is amazingly heterogeneous, and includes, among other things, topological and operator algebras, commutative and homological algebras, Lie theory, as well as rings and modules. His impact on algebra is though probably Kaplansky's most influential heritage, and he is remembered by his colleagues as “infinitely algebraic”: “I liked the algebraic way of looking at things. I’m additionally fascinated when the algebraic method is applied to infinite objects.” In this mostly historical memorial, we review Kap's mathematical legacy and its impact on the mathematics after him.

---

**Pere Ara**

## **Crossed products and the Atiyah problem**

It was shown by Austin in 2013 that the question by Atiyah about the rationality of  $H^2$ -Betti numbers has a negative answer. However the problem of determining the exact set of real numbers appearing as  $H^2$ -Betti numbers from a given group is widely open. In particular, this is an open question for the lamplighter group, which was the first known counterexample to the Strong Atiyah Conjecture, stating that all  $H^2$ -Betti numbers arising from a group belong to the subgroup  $\sum_{H \in G, H \text{ finite}} \frac{1}{|H|} \mathbb{Z}$  of  $\mathbb{R}$ .

I will review some recent progress on this question, obtained in joint work with Joan Claramunt and Ken Goodearl. I will recall some of the basic techniques in our approach, which involve the consideration of Sylvester matrix rank functions on certain crossed products, and their associate  $*$ -regular envelopes.

---

# $\pi$ -regular rings and periodic rings

Ayman BADAWI

**Abstract.** Let  $R$  be an associate ring with 1. Then  $R$  is called a  $\pi$ -regular ring if for every  $a \in R$ , there exist a positive integer  $n \geq 1$  and  $b \in R$  such that  $a^n b a^n = a^n$ . A ring  $R$  is called periodic if for every  $a \in R$ , there exist positive integers  $0 < n < m$  such that  $a^n = a^m$ . In this talk, we will explore some properties of  $\pi$ -rings and periodic-rings.

---

abadawi@aus.edu

American University of Sharjah, Department of Mathematics, P.O. Box 26666,  
University City Sharjah, UAE

# A General Research Idea in Ring and Module Theory via Replacing $=$ with $\simeq$ and Modules of Atomic Types

Mahmood Behboodi

Department of Mathematical Sciences, Isfahan University of Technology  
P.O.Box : 84156-83111, Isfahan, Iran, and  
School of Mathematics, Institute for Research in Fundamental Sciences (IPM)  
P.O.Box : 19395-5746, Tehran, Iran  
[mbehbood@iut.ac.ir](mailto:mbehbood@iut.ac.ir)

## Abstract.

The research idea replacing " $=$ " with " $\simeq$ " has been introduced since 2011 by the speaker and his research group (including his Ph.D students and some of his colleagues in Isfahan University of Technology).

If we express an important and basic concept of algebra in which " $=$ " is used, then " $=$ " can be substituted for " $\simeq$ " and most likely the new concept is not the same as the previous one. Thus we will have a non trivial generalization from them (Because in algebra, a kind of " $\simeq$ " plays the role of " $=$ ").

With this general idea, new research concepts can be defined such as: virtually simple, virtually indecomposable; virtually uniform; virtually uniserial, virtually homo-uniserial, almost uniserial and prime uniserial, virtually semisimple, virtually pure semisimple, virtually serial, virtually homo-uniserial and virtually local modules. (see [31]-[55] for the definitions, terminology and main results) . Even "isonoeitherian modules" and "isoartinian modules" introduced and studied independently by Nazemian and Facchini (see [56] and [57]), are close to our school of research and have the same spirit and flavour as ours, due to Nazemian being one of our Ph.D students in Isfahan University of Technology.

The study of the structure of rings via decompositions of modules into the certain classes of modules such as, simple modules, cyclic modules, indecomposable modules and the others....., has a rather long history.

Over the last 100 years, numerous authors including: J.H.M. Wedderburn, (1907), E. Artin (1927), G. Köthe (1935), K. Åsano (1939), T. Nakayama (1941), I.S. Cohen and I. Kaplansky (1951), Yu. Drozd, D. Eisenbud, A. Facchini, L. Fuchs, A.W. Goldie, P. Griffith, H. Kupisch, I. Murase, P. Prihoda, G. Puninski, L. Salce, T. Shores, W.J. Lewis, R. Warfield, Zimmermann-Huisgen, and Wisbauer, have investigated rings over which each module (or each finitely generated module) is a direct sum of one of the following generalizations of one-dimensional vector spaces to modules:

**Simple, Cyclic, Indecomposable, Uniform, Uniserial, Homo-uniserial, Completely cyclic, Local, Square free, Co-cyclic, Cyclically presented and Hollow modules.**

We note that, there are several classical structure theorems via decompositions of modules into some of the above type of modules such as Wedderburn-Artin theorem.

Is it possible to provide a definition that all the above concepts are included in this common definition, and especially when the ring  $R$  is a division ring, these concepts coincide with 1-dimensional vector spaces? Fortunately, the answer is yes, and in this talk, we call these concepts "**Modules of Atomic Types**".

**Definition:** Let  $R$  be a ring and  $A$  be a class of left  $R$ -modules. We say that the class  $A$  is of **atomic-type** if it satisfies the following conditions:

- 1)  $A$  contains all simple modules.
- 2)  $A$  is closed under isomorphism.
- 3) If  $M \in A$  and  $\text{Ann}_R(M)$  is maximal as a left ideal of  $R$ , then  $M$  is a simple  $R$ -module.

As applications of this research project, we give some new classes of atomic-type modules and then we give several generalizations of some classical structure theorems in ring and module theory (see [31]-[57]).

## References:

- [1] G. Kothe, Verallgemeinerte abelsche gruppen mit hyperkomplexen operatoren ring, Math. Z. 39 (1935) 31-44.
- [2] I. S. Cohen, I. Kaplansky, Rings for which every module is a direct sum of cyclic modules, Math. Z. 54 (1951) 97-101.
- [3] M. Auslander, A functorial approach to representation theory, Repr. Of Algebras, LNM 944 (1982), 105-179.
- [4] C. M. Ringel, Report on the Brauer-Thrall conjectures, Representation Theory I, LNM 831, (1971) 104-136.
- [5] I. Kaplansky, Elementary divisors and modules. Trans. Amer. Math. Soc. 66 (1949) 464-491.
- [6] I. Kaplansky, Modules over Dedekind rings and valuation rings. Trans. Amer. Math. Soc. 72 (1952) 327-340.
- [7] R. S. Pierce, Modules over commutative regular rings. Mem. Amer. Math. Soc. 70 (1967).
- [8] W. Brandal, Almost maximal integral domains and finitely generated modules. Trans. Amer. Math. Soc. 183 (1973) 203-222.
- [9] W. Brandal and R. Wiegand, Reduced rings whose finitely generated modules decompose. Comm. Algebra 6 (1978) 195-201.
- [10] T. S. Shores and R. Wiegand, Rings whose finitely generated modules are direct sums of cyclics. J. Algebra 32 (1974) 152-172.
- [11] P. Vámos, The decomposition of finitely generated modules and fractionally self injective rings. J. London Math. Soc. 16 (1977) 209-220.
- [12] R. Wiegand and S. Wiegand, Commutative rings whose finitely generated modules are direct sums of cyclics. Abelian group theory (Proc. Second New Mexico State Univ. Conf., Las Cruces, N.M., 1976), pp. 406-423. Lecture Notes in Math., Vol. 616, Springer, Berlin, 1977.
- [13] W. Brandal, Commutative Rings Whose Finitely Generated Modules Decompose, Lecture Notes in Mathematics 723, Springer-Verlag, Berlin, 1979.
- [14] Warfield, Robert B. Jr. (1975), "Serial rings and finitely presented modules.", J. Algebra, 37: 187-222
- [15] Faith, Carl (1976), Algebra. II. Ring theory, Grundlehren der Mathematischen Wissenschaften, No. 191. Springer-Verlag.
- [17] Hazewinkel, Michiel ; Gubareni, Nadiya; Kirichenko, V. V. (2004), Algebras, rings and modules. Vol. 1., Kluwer Academic Publishers.
- [18] D. Eisenbud and P. Griffith, The Structure of Serial Rings, Pacific J. Math 36 (1) (1971), 109-121.
- [19] T. Nakayama, On Frobeniusean algebras. II, Ann. Math. (2) 42 (1941), 1-21.
- [20] L. A. Skornyakov, When are all modules serial, Mat. Z. 5 (1969), 173-182.
- [21] S.U. Chase, Direct products of modules, Trans. Amer. Math. Soc. 97 (1960), 457-473.
- [22] R.B. Warfield Jr., Rings whose modules have nice decompositions, Math. Z. 125 (1972), 187-192.
- [23] B. Zimmermann-Huisgen, Rings whose right modules are direct sums of indecomposable modules, Proc. Amer. Math. Soc. 77 (1979), no. 2, 191-197.
- [24] M. Auslander, Notes on representation theory of artin algebras, Brandeis Univ, Waltham, Mass., 1972.
- [25] C. M. Ringel and H. Tachikawa, QF-3 rings, J. Reine Angew. Math. 272 (1975), 49-72.
- [26] K. R. Fuller and I. Reiten, Note on rings of finite representation type and decomposition of modules, Proc. Amer. Math. Soc. 50 (1975), 92-94.
- [27] L. Gruson and C. U. Jensen, Deux applications de la notion de L-dimension, C. R. Acad. Sci. Paris Sér. A 282 (1976), 23-24.
- [28] M. Auslander, Representation theory of Artin Algebras II, Comm. Algebra 1, (1974), 269-310.
- [29] M. Auslander, Large Modules over Artin Algebras, Algebra, Topology and Category Theory (Eilenberg), Acad. Press, (1976), 1-17.
- [30] B. Zimmermann-Huisgen, Rings whose right modules are direct sums of indecomposable modules, Proc. A.M.S. 77 (1979) 191-197.
- [31] M. Behboodi, S. Heidari, Commutative rings whose proper ideals are serial. Algebr. Represent. Theory 20 (2017), no. 6, 1531-1544.
- [32] M. Behboodi, Z. Fazelpour, Noetherian rings whose modules are prime serial. Algebr. Represent. Theory 20 (2017), no. 1, 245-255.
- [33] M. Behboodi, Z. Fazelpour and M.R. Vedadi, Rings all of whose prime serial modules are serial, J. Algebra Appl 16 (2017), no. 10, 1750193, 20 pp..
- [34] M. Behboodi, A. Daneshvar, M. R. Vedadi, . Virtually semisimple modules and a generalization of the Wedderburn-Artin theorem. Comm. Algebra 46 (2018), no. 6, 2384-2395.
- [35] M. Behboodi, A. Daneshvar, M. R. Vedadi, . Several generalizations of the Wedderburn-Artin theorem with applications. Algebr. Represent. Theory 21 (2018), no. 6, 1333-1342.
- [36] M. Behboodi, A. Daneshvar, M. R. Vedadi, Structure of virtually semisimple modules over commutative rings. Commun. Algebra 48, No. 7, 2872-2882 (2020).
- [37] M. Behboodi, E. Bigdeli, Prime virtually semisimple modules and rings. Comm. Algebra 47 (2019), no. 10, 3995-4008.
- [38] M. Behboodi, S. Roointan-Isfahani, Almost uniserial rings and modules. J. Algebra 446 (2016), 176-187.(Received 14 March 2015 Communicated by Louis Rowen).
- [39] M. Behboodi, S. Roointan-Isfahani, Local rings whose modules are almost serial. J. Pure Appl. Algebra 223 (2019), no. 1, 369-379.
- [40] M. Behboodi, A. Moradzadeh-Dehkordi, M. Qourchi Nejadi, Virtually uniserial modules and rings. J. Algebra 549 (2020), 365-385.
- [41] M. Behboodi, A. Ghorbani and A. Moradzadeh-Dehkordi, Commutative Noetherian local rings whose ideals are direct sums of cyclic modules, Journal of Algebra 345 (2011) 257-265.
- [42] M. Behboodi, A. Moradzadeh-Dehkordi, On Commutative rings whose prime ideals are direct sum of cyclic, Arch. Math. (Brno) 48 (2012), 291-299
- [43] Z. Nazemian, A. Ghorbani, M. Behboodi, , Uniserial dimension of modules, Journal of Algebra 399 (2014), 894-903.
- [44] M., Behbood, S.H. Shojaee, Commutative local rings whose ideals are direct sum of cyclic modules, Algebr. Represent Theor. 17 (2014), 971-982. (ISI) Q1
- [45] M. Behboodi, A. Ghorbani, A. Moradzadeh-Dehkordi, S. H. Shojaee, On left Kothe rings and a generalization of the Kothe-Cohen-Kaplansky theorem, Proc. Amer. Math. Soc. 142 (2014), 2625-2631
- [46] M. Behboodi, G. Behboodi Eskandari, , Local duo rings whose finitely generated modules are direct sums of cyclics, Indian J. Pure Appl. Math. 46 (1) (2015), 59-72.

- [47] M. Behboodi, Sh. Asgari, Commutative rings whose proper ideals are direct sums of uniform modules. *Comm. Algebra* 46 (2018), no. 3, 1277–1286.
- [48] M. Behboodi, Sh. Askari, Commutative rings whose proper ideals are direct sums of uniform modules, *Comm. Algebra* (to appear).
- [49] M. Behboodi, Z. Fazelpour and M.R. Vedadi, Rings all of whose prime serial modules are serial, *J. Algebra Appl* 16 (2017), no.10, 1750193, 20 pp.
- [50] Behboodi, M.; Moradzadeh-Dehkordi, A.; Qourchi Nejadi, M. Virtually homo-uniserial modules and rings. *Commun. Algebra* 49, No. 9, 3837-3849 (2021).
- [51] M. Behboodi (Joint with G. Behboodi Eskandari), On rings over which every finitely generated module is a direct sum of cyclic modules, *Haceteppe J. Math. Statis.* 45 (2016), no. 5, 1335–1342. (ISI) Q4
- [52] M. Behboodi (Joint with S. Heidari), Commutative rings whose maximal ideals are direct sums of completely cyclic modules, *Mathematical Reports (Math. Rep. (Bucur.))* 18(68) (2016), no. 4, 609–617. . (ISI) Q4
- [53] M. Behboodi (Joint with S. Heidari and S. Roointan-Isfahani), Commutative rings whose proper ideals are direct sum of completely cyclic modules, *J. Algebra Appl.* 15 (2016), # 12 Pages (ISI)
- [54] M. Behboodi, A. Daneshvar, M. R. Vedadi, Virtually Semisimple Modules and a Generalization of the Wedderburn-Artin Theorem, arXiv:1603.05647v1, [Submitted on 17 Mar 2016 (this version), latest version 31 May 2016 (v2)]
- [55] M. Behboodi, A. Daneshvar, M. R. Vedadi, Two Generalizations of the Wedderburn-Artin Theorem with Applications, arXiv:1605.09740v1, [Submitted on 31 May 2016 (v1), last revised 15 Oct 2016 (this version, v3)]
- [56] Facchini, A.; Nazemian, Z., Modules with chain conditions up to isomorphism, *J. Algebra*, 453, 578-601, (2016).
- [57] Facchini, A.; Nazemian, Z., On isonoetherian and isoartinian modules. (English) *Zbl* 1423.16004 Facchini, Alberto (ed.) et al., Model theory of modules, algebras and categories. International conference, Ettore Majorana Foundation and Centre for Scientific Culture, Erice, Italy, July 28 – August 2, 2017. Providence, RI: American Mathematical Society (AMS). *Contemp. Math.* 730, 1-22 (2019).

## Gabriella D'Este

More or less new results on tilting and tau tilting modules

---

### ABSTRACT

In the first part of my talk I will give a visual presentation of tilting modules in the classical sense of S. Brenner and M.C.R. Butler [ 1 ]. These modules are defined over finite dimensional algebras and are vector spaces of finite dimension. Moreover they have projective dimension at most one, no selfextension and - up to multiplicity - the number of their indecomposable summands is equal to the number of simple modules. In the second part of I will describe some results contained in my joint paper with H. Melis Tekin Akcin [ 2 ] and some work in progress.

- [ 1 ] S. Brenner and M.C.R. Butler, Generalizations of the Bernstein - Gelfand - Ponomarev reflections functors. Springer LNM 832 (1979) , 103 - 169 .
- [ 2 ] G. D'Este and H.M. Tekin Akcin, A bijection between the indecomposable summands of two multiplicity free tilting modules, available with Google on ArXiv and published on 22 June 2021 in the Bulletin of the Iranian Mathematical Society,



# AN ALGEBRAIC APPROACH TO OPERATOR THEORY

G. H. ESSLAMZADEH

Our main goal here is to show that many of essential results in operator theory rely on the algebraic structure of the unital ring  $\mathcal{B}(\mathcal{H})$  of bounded operators on a Hilbert space  $\mathcal{H}$ . The wide spectrum of structures on this ring is the main motivation for investigating the role of algebraic structure of  $\mathcal{B}(\mathcal{H})$  in different major results in operator theory and operator algebras. Our strategy for dealing with this general problem is finding the right category, containing operator algebras, in which a specific result of operator theory remains true. We approach this problem from three directions.

First, we work in the much larger category of unital  $*$ -algebras. We show that the matricial structure of operator algebras, which is one of the main tools in quantized functional analysis, is largely kept in this category. In particular algebraic extensions of Choi-Effros characterization of operator systems, Ruan's theorem and Arveson's extension theorem were proved.

In the second approach we keep just the lattice structure of projections of  $\mathcal{B}(\mathcal{H})$  and loose the other structures. More precisely we choose to work in the category of Baer  $*$ -rings where a wealth of projections exist. In this part we focus on the major decomposition theorems. We present algebraic analogs of Wold, Nagy-Foias-Langer and Halmos-Wallen decomposition theorems for isometries, contractions and power partial isometries respectively. Then extensions to simultaneous decompositions for certain sets of operators with cardinality of bigger than one, such as commuting  $n$ -tuples of isometries and semigroups of isometries are discussed.

The third approach which is ongoing research, is devoted to investigating existence of projections properties in the category of  $*$ -algebras. So far we have established some characterizations of Rickart  $C^*$ -algebras, certain type I  $C^*$ -algebras and group algebras, in terms of topological properties.

DEPARTMENT OF MATHEMATICS, SHIRAZ UNIVERSITY, IRAN  
*E-mail address:* `esslamz@shirazu.ac.ir`

## **Alberto Facchini**

title: Multiplicative lattices, groups, braces

Abstract: The multiplicative lattices we will consider are those defined in [Facchini, Finocchiaro and Janelidze, Abstractly constructed prime spectra, *Algebra universalis* 83(1) (February 2022)]. Multiplicative lattices yield the natural setting in which several basic mathematical questions concerning algebraic structures find their answer. We will consider the particular cases of groups [Facchini, de Giovanni and Trombetti, *Spectra of groups*, to appear, 2021] and braces [Facchini, *Algebraic structures from the point of view of complete multiplicative lattice*, to appear, 2022, <http://arxiv.org/abs/2201.03295>].

# Torsion free modules over commutative domains

Dolors Herbera  
Departament de Matemàtiques,  
Universitat Autònoma de Barcelona,

Centre de Recerca Matemàtica,  
08193 Bellaterra (Barcelona), Spain  
e-mail: dolors@mat.uab.es

Let  $R$  be a commutative domain. Let  $\mathcal{F}$  be the class of  $R$  modules that are infinite direct sums of finitely generated torsion-free modules. In the talk we will discuss the question whether  $\mathcal{F}$  is closed under direct summands.

If  $R$  is local of Krull dimension 1,  $\mathcal{F}$  being closed under direct summands is equivalent to say that any indecomposable, finitely generated torsion-free module has local endomorphism ring.

If  $R$  is an  $h$ -local domain, then we show that the property  $\mathcal{F}$  is inherited by the localization at a maximal ideal. Moreover, there is an interesting relation between ranks of indecomposable modules over such localizations.

To prove such results we use two main tools:

- Příhoda's theory of fair-sized projective modules [3, 2] and its extension to a non-noetherian setting [1], that will give us a way to construct infinitely generated non-trivial summand,
- an extension of the Package Deal Theorems by Levy Odenthal, to the setting of  $h$ -local domains [4].

## References

- [1] Herbera, D., & Příhoda, P. *Reconstructing projective modules from its trace ideal*. Journal of Algebra, **416** (2014), pp. 25–57.
- [2] Herbera, D., Příhoda, P. and Wiegand, R. *Big pure-projective modules over commutative noetherian rings: comparison with the completion*. Preprint (2022).
- [3] P. Příhoda, *Fair-sized projective modules* Rend. Semin. Mat. Univ. Padova **123** (2010), 141–167.

- [4] Levy, L. S., Odenthal, C. J. . *Package Deal Theorems and Splitting Orders in Dimension 1*. Transactions of the American Mathematical Society, **348** (9) (1996), 3457–3503.

**Omid Ali Shahni Karamzadeh**

My suggested title is: How did I turn up doing non-commutative ring theory for my Ph.D., in 1971, at Exeter University, under a world-famous commutative algebraist, and what are some of my achievements in this regard, from my own point of view?

**Arezou Karimi Mansoub**

The title of my talk is:

Fully Homomorphic encryption using ideal lattices

# On the Application of Algebraic Number Theory in Wireless Communication

Hassan Khodaiemehr  
Department of Computer Science and Statistics,  
Faculty of Mathematics  
K. N. Toosi University of Technology

**Abstract:** Algebraic techniques have been applied in situations where one, a priori, would not expect them at all. Finite fields are key tools in the mathematical background of digital communications and in the design of powerful binary and non-binary codes. Due to the technological developments and increased processing power of digital receivers, attention moved to the design of signal space codes in the framework of coded modulation systems. Here, the theory of Euclidean lattices became of great interest for the design of dense signal constellations well suited for transmission over the additive white Gaussian noise (AWGN) channel.

More recently, the incredible developments of wireless communications forced coding theorists to deal with fading channels. New code design criteria had to be considered in order to improve the poor performance of wireless transmission systems. The need for bandwidth efficient coded modulation became even more important due to the scarce availability of radio bands. Algebraic number theory was shown to be a very useful mathematical tool that enables the design of good coding schemes for fading channels. These codes are constructed as multidimensional lattice signal constellations where highest coding gain is obtained by introducing the so-called modulation diversity (or signal space diversity) in the signal set. This results in a particular type of bandwidth-efficient diversity technique.

There are two approaches to construct high modulation diversity constellations. The first is based on the design of intrinsic high diversity algebraic lattices, obtained by applying the canonical embedding of an algebraic number field to its ring of integers. The second approach to obtain high modulation diversity is based on applying a particular rotation to a multidimensional quadrature amplitude modulation (QAM) signal constellation in such a way that any two points achieve the maximum number of distinct components. The results of this method for achieving diversity can also be obtained using algebraic number theory. In this talk we focus more on such applications.

M. Tamer KOSAN

The title of my talk is On rings determined by their idempotents and units (or nilpotents)



André Leroy

Evaluations of iterated skew polynomials and  
some of its consequences.

The title is " Weakly Principally Quasi-Baer Rings and their Extensions"

Ryszard Mazurek, Bialystok University of Technology, Poland

Title: Around chain rings

Abstract: A ring  $R$  is called a right (left) chain ring if its right (left) ideals are linearly ordered by inclusion, and  $R$  is called a chain ring if it is a right and left chain ring. Chain rings are a natural generalization of valuation rings. In this talk, I will give an overview of results on (one-sided) chain rings and related structures (e.g., rings with linearly ordered right annihilators, rings with a distributive lattice of right ideals, semigroups with linearly ordered right ideals), and on some connections of chain rings with the theory of radicals.

**Diego Napp UA**

## **Title:** Smaller Keys for Code-Based Cryptography: McEliece Cryptosystems with Convolutional Encoders

**Abstract:** In this talk I will first present the state of the art of Post-Quantum (code-based) cryptography. I will then introduce a new variant of the code-based Public Key Cryptosystem of McEliece. This novel cryptosystem possesses several interesting properties, including a significant reduction of the public key for a given security level. In contrast to the classical McEliece cryptosystems, where block codes are used, we propose the use of a convolutional encoder to be part of the public key. The secret key is constituted by a Generalized Reed-Solomon encoder and two Laurent polynomial matrices that contain large parts that are generated completely at random. In this setting the message is a sequence of messages instead of a single block message and the errors are added randomly throughout the sequence. I will conclude the talk by studying its security and presenting several examples.

Kamal Paykan

Title: Some new decompositions in the Jacobson radicals  
of well-known ring dxtensions

Marjan Sheibani

**Title: Generalized inverses in rings and Banach algebras**

# ON SOME FACTORIZATION QUESTIONS IN NONCOMMUTATIVE NOETHERIAN DOMAINS

DANIEL SMERTNIG

ABSTRACT. Over the last decades, the study of factorizations of elements in commutative domains and monoids (in particular, Krull and Mori domains and monoids) has emerged as a subfield of algebra, number theory, and additive combinatorics. The last years have seen an effort to extend this theory to a noncommutative setting. In some cases smooth results are available also in the noncommutative setting (e.g., for bounded hereditary noetherian prime rings that are also Hermite rings), but in general even some basic finiteness results remain elusive.

For instance, a domain  $R$  is a bounded factorization (BF) domain if for every  $0 \neq a \in R$ , there exists  $\lambda(a) \in \mathbb{N}_0$  such every factorization of  $a$  into atoms (irreducible elements) has length at most  $\lambda(a)$ . It is well-known that commutative noetherian domains are BF-domains (more generally, this holds for Mori domains and monoids). No such comprehensive result is available for noncommutative domains. We present an overview on factorizations in noncommutative rings and recent progress on sufficient conditions using a variety of methods, such as filtrations, reduced rank, growth-related properties, and homological methods.

*Parts of the talk refer to joint work with Jason Bell, Ken Brown, and Zahra Nazemian.*

Ashish Srivastava

**Title:** Leavitt path algebras and factorization theory of their ideals

**Abstract:** In this talk we will discuss Leavitt path algebras and factorization of their ideals in terms of various special types of ideals.



Tapkin Dan Tagirzyanovich

Title: Rings close to clean

# RINGS WHOSE SINGULAR CYCLIC MODULES ARE ARTINIAN

M. R. VEDADI

ABSTRACT. A widely used result of C. Hopkins and J. Levitzki states that “every right Artinian ring is a right Noetherian ring” (1939). Various generalizations of this theorem have been given by the many authors. In 1972, V. Camillo and G. Krause asked: Is a ring  $R$  right Noetherian if every proper cyclic right  $R$ -module is Artinian?. It is easily seen that rings with latter property are Artinian or have 1 uniform dimension (i.e. non-zero right ideals in  $R$  are essential). Clearly,  $I$  is an essential right ideal of  $R$  iff the cyclic right  $R$ -module  $R/I$  singular. In this lecture, recent results on rings with right restricted minimum condition (r.RMC, for short) are explained. Here, r.RMC means singular cyclic right modules are Artinian. In particular, answers to the following two questions are provided: (i) Is a left self-injective ring with r.RMC quasi-Frobenius? (ii) Whether a serial ring with r.RMC must be Noetherian?

*E-mail address:* `mrvedadi@cc.iut.ac.ir`

DEPARTMENT OF MATHEMATICAL SCIENCES, ISFAHAN UNIVERSITY OF TECHNOLOGY, 84156-83111, ISFAHAN, IRAN

Title: Linear Codes over Finite Frobenius Rings

Speaker: **Jay A. Wood**, Western Michigan University

Abstract: The 1962 doctoral dissertation of F. J. MacWilliams contains two theorems that have proven to be foundational in algebraic coding theory: an extension theorem that shows that two notions of equivalence for linear codes are the same, and the McWilliams identities that relate the Hamming weight enumerators of a linear code and its dual. This talk will be a survey of how these theorems have been generalized to the context of linear codes defined over finite Frobenius rings and over modules with cyclic socles.

---

**Mohamed F. Yousif**  
The Ohio State University  
yousif.1@osu.edu

International Conference on Noncommutative Algebra and its Applications  
Tarbiat Modares University  
Tehran, Iran  
May 9 - 12, 2022

## Modules with the Exchange Property

**Abstract:** A right  $R$ -module  $M$  is said to satisfy the (full) exchange property if for any two direct sum decompositions  $L = M \oplus N = \bigoplus_{i \in I} N_i$ , there exist submodules  $K_i \subseteq N_i$  such that  $L = M \oplus (\bigoplus_{i \in I} K_i)$ . If this holds only for  $|I| < \infty$ , then  $M$  is said to satisfy the finite exchange property. The exchange property is of importance because it provides a way to build isomorphic refinements of different direct sum decompositions, which is precisely what is needed to prove the famous Krull-Schmidt-Remak-Azumaya Theorem. It is an open question due to Crawley and Jónsson whether the finite exchange property always implies the full exchange property.

In this talk we present the latest results on this open question and its relationship with clean rings and modules. This is a joint work with Yasser Ibrahim of both Taibah and Cairo Universities.